Exercise 322

Solve the following trigonometric equations on the interval $\theta = [-2\pi, 2\pi]$ exactly.

$$6\cos^2 x - 3 = 0$$

Solution

Isolate the term with x.

$$6\cos^2 x = 3$$

Divide both sides by 6.

$$\cos^2 x = \frac{1}{2}$$

Take the square root of both sides.

$$\sqrt{\cos^2 x} = \sqrt{\frac{1}{2}}$$

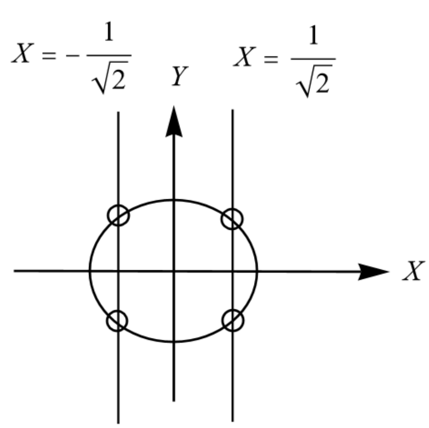
If there's an even power under an even root and the result has an odd power, it needs an absolute value sign.

$$|\cos x| = \frac{1}{\sqrt{2}}$$

Remove the absolute value sign by placing $\pm$ on the right side.

$$\cos x = \pm\frac{1}{\sqrt{2}}$$

Cosine represents the horizontal distance on the unit circle.



The two vertical lines go through the unit circle in four locations. The value of x at the top-right location is

$$x = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4},$$

and the value of x at the top-left location is

$$x = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}.$$

Add π to the value of x at the top-right location to get the value of x at the bottom-left location.

$$x = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$$

Add π to the value of x at the top-left location to get the value of x at the bottom-right location.

$$x = \frac{3\pi}{4} + \pi = \frac{7\pi}{4}$$

Subtract 2π from all these values of x to get the values between $[-2\pi, 0]$.

$$\frac{\pi}{4} - 2\pi = -\frac{7\pi}{4}$$

$$\frac{3\pi}{4} - 2\pi = -\frac{5\pi}{4}$$

$$\frac{5\pi}{4} - 2\pi = -\frac{3\pi}{4}$$

$$\frac{7\pi}{4} - 2\pi = -\frac{\pi}{4}$$

Therefore,

$$x = \left\{-\frac{7\pi}{4}, -\frac{5\pi}{4}, -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}.$$